Math 45 SSM 2/e 5.3 Multiplying Polynomials

Objectives
1) multiply monomial by polynomial using distribute
2) multiply binomial by binomial using FOIL
3) special products: recognize patterns for difference of squares and perfect square trinomials.
4) multiply polynomial × polynomial using term-by-term distribution.

Multiply.

① $-2xy(3x^2+5xy+2y^2)$ mono × polynomial

$= -2xy \cdot 3x^2 + (-2xy)(5xy) + (-2xy)(2y^2)$ dist $-2xy$

$= \boxed{-6x^3y - 10x^2y^2 - 4xy^3}$ multiply monomials.

② $\left(\frac{4}{3}z^2 + 8z + \frac{1}{4}\right)\frac{1}{2}z^3$ poly × monomial

$= \frac{1}{2}z^3\left(\frac{4}{3}z^2 + 8z + \frac{1}{4}\right)$ dist $\frac{1}{2}z^3$

$= \left(\frac{1}{2}z^3\right)\left(\frac{4}{3}z^2\right) + \left(\frac{1}{2}z^3\right)(8z) + \left(\frac{1}{2}z^3\right)\left(\frac{1}{4}\right)$

$= \frac{1}{\cancel{2}}\cdot\frac{\overset{2}{\cancel{4}}}{3}z^{3+2} + \frac{1}{\cancel{2}}\cdot\overset{4}{\cancel{8}}z^{3+1} + \frac{1}{2}\cdot\frac{1}{4}z^3$

$= \boxed{\frac{2}{3}z^5 + 4z^4 + \frac{1}{8}z^3}$

③ $(x+3)(x-8)$ binomial × binomial

Method 1:

$$\begin{array}{rrrr} & & x & +3 \\ \times & & x & -8 \\ \hline & & -8x & -24 \\ + & x^2 & +3x & \\ \hline & x^2 & -5x & -24 \end{array}$$

vertical method same as

$$\begin{array}{rr} & 21 \\ \times & 31 \\ \hline & 21 \\ + & 63\ \ \\ \hline & 651 \end{array}$$

(3) $(x+3)(x-8)$

Method 2: F O I L ← This is an acronym. Each letter stands for a word.

F = first
O = Outer
I = inner
L = last

$(x+3)(x-8)$

F = First times First → x^2

$(x+3)(x-8)$

O = outer → $-8x$

$(x+3)(x-8)$

I = inner → $3x$

$(x+3)(x-8)$

L = Last → -24

$x^2 - 8x + 3x - 24$

$\boxed{x^2 - 5x - 24}$

(4) $(3x+4)(2x-5)$

$= 3x \cdot 2x - 3x \cdot 5 + 4 \cdot 2x + 4(-5)$ FOIL

$= 6x^2 - 15x + 8x - 20$ multiply

$= \boxed{6x^2 - 7x - 20}$ combine like terms

(5) $(3a+1)(a-4)$

$= 3a^2 - 12a + a - 4$ FOIL & mult

$= \boxed{3a^2 - 11a - 4}$ Combine

(6) $(x-5)(x+5)$

$= x^2 + 5x - 5x - 25$ FoIL & mult

$= \boxed{x^2 - 25}$ Combine

square difference square

This result is called a "<u>difference of two squares</u>."

The difference of two squares pattern (or just "difference of squares") happens when the two binomials have same terms except signs:

$$(a+b)(a-b) = a^2 - b^2$$

We will use this a lot in chapter 6.

(7) $(4x-3y)(4x+3y)$

$= 16x^2 + 12xy - 12xy - 9y^2$

$= \boxed{16x^2 - 9y^2}$ ← another difference of two squares.

(8) $(3-r)^2$

$= (3-r)(3-r)$ This exponent is on base $(3-r)$.

$= 9 - 3r - 3r + r^2$

$= 9 - 6r + r^2$

$= \boxed{r^2 - 6r + 9}$ ← This pattern is called a "perfect square trinomial" because it begins with a square (exp 2) and ends with 3 terms (trinomial). We'll use this a lot in chapter 6.

$$\boxed{\begin{aligned}(a+b)^2 &= a^2 + 2ab + b^2\\ (a-b)^2 &= a^2 - 2ab + b^2\end{aligned}}$$

(9) $(2x+5y)^2$

$= (2x+5y)(2x+5y)$

$= 4x^2 + 10xy + 10xy + 25y^2$

$= \boxed{4x^2 + 20xy + 25y^2}$ ← another perfect square trinomial

(10) $(x+1)^2 - (2x+1)(x-1)$

$= (x+1)(x+1) - (2x+1)(x-1)$ ← order of operations: exp first

$= (x^2+x+x+1) - (2x^2-2x+x-1)$ ← multiply next

$= (x^2+2x+1) - (2x^2-x-1)$ ← combine like terms

$= x^2 + 2x + 1 - 2x^2 + x + 1$ ← subtract polynomials

$= \boxed{-x^2 + 3x + 2}$

(11) $-\frac{1}{2}x\underbrace{(2x+6)(x-3)}$

Commutative property says we can multiply in any order. Let's do the fraction last.

$= -\frac{1}{2}x\left[2x^2-6x+6x-18\right]$

$= -\frac{1}{2}x\left[2x^2-18\right]$ FOIL, combine like terms

$= -\frac{1}{2}x\cdot(2x^2)-\left(\frac{1}{2}x\right)(-18)$ dist $\left(-\frac{1}{2}x\right)$

$= \boxed{-x^3+9x}$

(12) $(a+6b)^2-(a-6b)^2$

$= (a+6b)(a+6b)-(a-6b)(a-6b)$ exponents first

$= (a^2+6ab+6ab+36b^2)-(a^2-6ab-6ab+36b^2)$ mult

$= (a^2+12ab+36b^2)-(a^2-12ab+36b^2)$ combine

$$= \begin{array}{r} a^2+12ab+36b^2 \\ -a^2+12ab-36b^2 \\ \hline \end{array}$$ ← dist neg

$= \boxed{24ab}$ combine like terms.

dist 3

(13) $(2x+3)(x^2+5x-1)$

dist 2x

This problem can also be organized using
- vertical method
- boxes

$$= \begin{array}{r} 2x^3+10x^2-2x \quad\quad \\ +3x^2+15x-3 \\ \hline \end{array}$$

$= \boxed{2x^3+13x^2+13x-3}$

(13) By boxes method

	x^2	$+5x$	-1
$2x$	$2x^3$	$10x^2$	$-2x$
$+3$	$3x^2$	$15x$	-3

⇐ one polynomial across top separated one term per box

⇑ one polynomial down left separated one term per box

multiply each.
combine like terms

$$\boxed{2x^3+13x^2+13x-3}$$

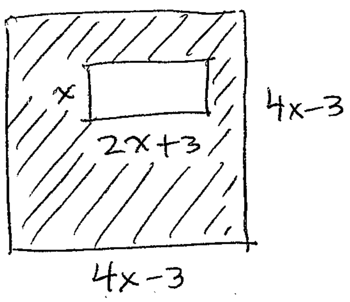
(13) By vertical method

$$\begin{array}{rrrr} & x^2 & +5x & -1 \\ \times & & 2x & +3 \\ \hline & +3x^2 & 15x & -3 \\ 2x^3 & +10x^2 & -2x & \\ \hline 2x^3 & +13x^2 & +13x & -3 \end{array}$$

multiply term by term

$\Rightarrow$ $\boxed{2x^3+13x^2+13x-3}$

(14) Find the area of the shaded region.



Area of shaded region = $\begin{pmatrix}\text{area of}\\ \text{outside}\\ \text{box}\end{pmatrix} - \begin{pmatrix}\text{area of}\\ \text{inside}\\ \text{box}\end{pmatrix}$

Area each box = length · width.

Shaded $= (4x-3)(4x-3) - x(2x+3)$

$= 16x^2-12x-12x+9 \quad -2x^2-3x$ FOIL & dist

$= \boxed{14x^2-27x+9}$ units^2.

5.3.111 Perform the indicated operation.

$$6x^2(x+6) - 8x(x^2-1)$$

$6x^2(x+6) - 8x(x^2-1) =$

(Simplify your answer. Type your answer in standard form.)

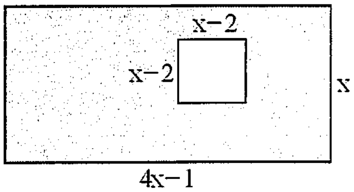
distribute (multiply) before subtract.

$$6x^2(x+6) - 8x(x^2-1)$$

$$= 6x^3 + 36x^2 - 8x^3 + 8x$$

$$= \boxed{-2x^3 + 36x^2 + 8x}$$

5.3.131 Find an algebraic expression that represents the area of the shaded region.



The area of the shaded region is . (Simplify your answer. Do not factor.)

$$\begin{pmatrix}\text{Area}\\ \text{shaded}\end{pmatrix} = \begin{pmatrix}\text{Area of}\\ \text{large box}\end{pmatrix} - \begin{pmatrix}\text{Area of}\\ \text{Small box}\end{pmatrix}$$

$$= (\text{length})\cdot(\text{width}) - (\text{length})\cdot(\text{width})$$

$$= (4x-1)(x) - (x-2)(x-2)$$

$$= 4x^2 - x - (x^2 - 2x - 2x + 4) \qquad \text{dist FOIL}$$

$$= 4x^2 - x - (x^2 - 4x + 4) \qquad \text{Combine}$$

$$= 4x^2 - x - x^2 + 4x - 4$$

$$= \boxed{3x^2 + 3x - 4}$$